

# A Dimensional View toward Vagueness\*

No name for blind review

**Abstract**—This paper suggests a new understanding toward vagueness by proposing a new formal semantics. This semantics imports the formal concept of dimensions, which simulates absence and abundance of information. This formalization provides not only a working framework which accounts for our linguistic activities but also an unified view which combines the existing accounts towards this puzzling concept.

## VAGUENESS REVISITED

Most (or perhaps all) verbal expressions are vague. We allow and accept a lot of vagueness in our daily life communication. When we talk about properties like baldness (via adjectives “be bald”), we seem to adopt (seemingly) conflicting intuitions: We should not take a sharp threshold between something is P and something is non-P (i.e. there is no rigid number which determines whether or not something is bald) while we can say that something (e.g. a person with *no* hair at all) is definitely P.

A classical example is a heap (or *soros*, in Greek): given a heap of sands and let us move just a single piece of sand from it. We take granted that such a small change does not make it to stop being a heap (*tolerance* principle). Repeating this removing, however, leads to a counter-intuitive consequence: even the last piece (or even none) of sand is still counted as a heap. Vagueness arising in this context has been discussed in philosophy or philosophical logic (cf. [KS97]) for its risk to cause a paradox, known as *sorites paradox* (cf. [Hyd11], [Sai95, Ch.3]).

## DIRECTIONS SUGGESTED

Many solutions have been suggested towards this problem. They can be categorized into the two groups: *logical* and *non-logical* [HR18]. *Logical* solutions think that there is something wrong in the reasoning which derives from (seemingly) adequate assumptions to the problematic conclusion.

Philosophical logicians often suggested to reconsider classical logic. On the other hand, non-logical solutions criticize some of the assumptions.

There is a third direction, which the argument of this article ultimately belongs to: not to solve but to *embrace* the paradox. This direction find neither assumptions nor reasoning systems problematic. Rather, it advises to embrace this paradox for it reflects some aspects of the nature of languages (semantic vagueness) or even metaphysical reality (cf. ontic vagueness). More particularly, this article proposes a formal description of such reality that the sorites paradox happens due to either semantical or ontic vague nature of the structure we live in.

## A FORMALIZATION SUGGESTED (BY ME)

The key feature of my semantics is the concept of dimensions. This formal component mocks *absence or abundance of information*, which causes vagueness. Borderline cases between P and non-P occur because we have only insufficient information (too less information; lack or absence of information) or unnecessary and confusing information (too much information; abundance of information). A model case of dimensions at work for vagueness, which would be the most familiar to most readers, is our vision. When we see things in the three (or more)-dimensional space (as naively believed, setting aside the (meta)physical question), we are not grasping them directly in the three dimensional structure. Rather, we see things via our retina, which themselves are two-dimensional and perceive them in the two dimensional structure. This fact of having information constrained by our biological with dimensions less than the reality provides us a chance to disagree about the same thing: A cylinder looks a circle from my perspective while it looks a rectangle from your perspective and neither from her perspective.

Technically speaking, absence and abundance of information are written in terms of *projection functions*. Given a dimensional structure, say a product of (topological) spaces  $\prod_n \langle X_i, \tau_i \rangle$ , a projection function  $f : \prod_n \langle X_i, \tau_i \rangle \mapsto \prod_{n-m} \langle X_i, \tau_i \rangle$  returns a structure with a less ( $-m$ ) number of dimensions.

#### SUGGESTIONS CONNECTED

After introducing formal components of my semantics, I will demonstrate connections lying between my dimensional understanding and the previous attempts to this paradox. One of the main merits of my semantics is to offer a formal *platform* for existing (and sometimes conflicting) explanations. In other words, this formalization several variants, which describe each philosophical solution and philosophical disagreement itself (i.e. why and how they disagree with each other).

For one thing, *supervaluationists* such as Fine followed by Keefe [Kee00] would adopt this semantics because imposing dimensions makes possible to describe semantic differences which they expressed by the operator D (read “definitely”). Their formal desiderata of truth value *gaps* are well written in our dimensional framework either by having too much information which conflict within itself or too less information which does not offer enough evidence. In both cases, we hesitate to assign a rigid truth value.

Interestingly, one of their rival *epistemicists* such as Williamson [Wil94] would also accept my semantics as a natural formalization on their epistemic account, which claims that vagueness is due to our epistemic shortage or *ignorance* that we do not know the sharp threshold between P and non-P. In my dimensional picture, information given to us is limited in number of dimensions.

Furthermore, I will discuss technical connections to other spatial formalizations, especially ones featuring the idea of *conceptual space* [Gär00] [Gär14] such as [Dou+13].

If time and interests of the audience permit, I will discuss a broader picture behind this vagueness via dimensions project: to combine metaphysical theories of modality via dimensions (cf. Yagisawa’s modal dimensionalism [Yag10]).

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