A Dimensional View towards Vagueness ⋆

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Abstract. This paper suggests a new understanding of *vagueness* by proposing a new formal semantics. This semantics imports the formal concept of *dimensions*, which simulates two sources of vagueness: *absence* and *abundance* of information. This formalization provides not only a working framework which accounts for our linguistic activities but also a unified framework which combines the existing accounts towards this puzzling concept.

Keywords: vagueness · sorites paradox · semantics

1 INTRODUCTION

Most, or perhaps all, verbal expressions are *vague*. We allow and accept a lot of vagueness in our daily life communication. Better: we do not usually have a complete full agreement towards the meaning (and/or truth conditions) for expressions. Vagueness does not always its bad, but sometimes does. When we talk about properties like baldness (via adjectives "be bald"), we seem to adopt seemingly conflicting intuitions as follows: (i) We should not take a sharp threshold between something is P and something is non-P (i.e. there is no rigid number which determines whether or not something is bald), but at the same time, (ii) we can say that something (e.g. a person with *no* hair at all) is definitely P.

A classical example is a heap (or *soros* in Greek): given a heap of sands and let us move just a single piece of sand from it. We take granted that such a small change does not make it stop being a heap–this assumption has been called *tolerance* principle in the literature. Repeating this removing, however, leads to a counter-intuitive consequence: even the last piece (or even none) of sand is still counted as a heap. Vagueness arising in this context has been discussed in philosophy or philosophical logic (cf. [4]) for its risk to cause a paradox, known as *sorites paradox* (cf. [1], [5, Ch.3]).

Towards this problem, many solutions and approaches have been suggested. According to [2], the solutions can be categorized into the two groups: *logical* and *non-logical*. *Logical* solutions think that there is something wrong in the reasoning which derives from adequate assumptions to the problematic conclusion. Philosophical logicians often suggested to reconsider and revise classical logic. For instance, Kit Fine suggested a popular solution labeled *supervaluationist*

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approach. In a nutshell, this approach allows *gaps* of truth value, saying that some object x is *neither* $P(x)$ nor $\neg P(x)$ for a vague predicate P. This refuses what classical logic forces us: law of bivalence $\phi \vee \neg\phi$.

On the other hand, *non-logical* solutions criticize some of the assumptions in the paradox. The most typical and popular instance of this direction would be *epistemicists'* approach (supported by [6]). Epistemists discard the *tolerance* principle and insists that there *exists* a sharp threshold separating which is bald and which is not. However, they claim, we are ignorant to notice such a cutting point: we often face the sorites paradox because we are *epistemically* insufficient to find where the thresholds are.

Furthermore, there is a third direction: not to solve but to *embrace* the paradox. This direction find neither assumptions nor reasoning systems problematic. Rather, it advises to embrace this paradox for it reflects some aspects of the nature of languages (semantic vagueness) or even metaphysical reality (cf. ontic vagueness).

Despites many varied options available in the market, they seem to share the single target. They all should aim at capturing the same target concept: *vagueness*. That says, even though their answers may be all different but what these answers are trying to answer is not different concepts vaguenesses but the same concept vagueness. *We just disagree with each other about the same vagueness. We do not talk about different vaguenesses.* Considering this point, this paper does *not* argue which option is right nor which is better than others. The goal of this paper is not to argue which philosophical solution works better nor to suggest a new alternative to destroy the paradox. Rather, the aim is to offer an unified formal semantics which can formally express not only the mechanism working behind sorites paradoxes but also philosophical ideas suggested so far.

The construction of this paper is as follows. In the next section, my formalization will be presented, followed by several intuitive examples. After that, the sorites paradox is interpreted in my dimensional framework. The final section points out several connections between my suggestion and other previous thoughts, providing particualr examples of the merit of my formal framework.

2 A FORMALIZATION SUGGESTED (BY ME)

The key feature of my semantics is the concept of *dimensions*. This formal component mocks *absence* or *abundance* of information, which causes vagueness according to my diagnosis. Borderline cases between P and non-P occur because we have insufficient and poor information (i.e. *too little* information; lack or absence of information) or unnecessary and confusing information (i.e. *too much* information; abundance of information).

2.1 Intuitive explanation

Before the formal definitions, I will offer an intuitive grasp on this dimensional idea by examples. A model case of dimensions at work for vagueness, which would be the most familiar to most readers, is our *vision*. When we see things in the three (or more)-dimensional space (as naively believed, setting aside the (meta)physical question), we are not grasping them directly in the three dimensional structure. Rather, we see things via our retina, which themselves are two-dimensional and perceive them in the two dimensional structure. This fact of having information constrained by our biological with dimensions less than the reality provides us a chance to disagree about the same thing. For instance, a cylinder looks a circle from my perspective while it looks a rectangle from your perspective and neither from her perspective. The answer towards a question "What is this object?" is reasonably *vague* from a limited number of dimensions like me, you, and her with only (her own) two-dimensional structure. However, imagine a four-dimensional creature whose retina captures things directly in three-dimensional structure. For the creature, the cylinder is obviously a cylinder and not vague. Why? My dimensinoal view says because the creature has a more refined information, dimensinoally speaking, with many more dimensions. More realistically, consider another person who collects the answers of me, you and her. If she succeeds to construct a cylinder out of information composed of segmented and limited information provided by the other three, she is more sure about what the thing is.

2.2 Formalization

What I have just said is formally expressed in what follows. Technically speaking, absence and abundance of information are written in terms of *projection functions*. Given a dimensional structure, say a product of spaces $\prod_{i \in I} X_i$, a projection function $f : \prod_{i \in I} X_i \mapsto \prod_{j \in J} X_j$ returns a structure with a less number of dimensions (take $|J| \leq |I|$). To begin with, we build a dimensional structure to evaluate vague (and other) predicates.

Definition 1 (Dimensional structure). *Let X_i a space (set). A dimensional structure M is defined as follows:*

$$M^i = \prod_{i \in I} X_i.$$

Upon this formal base, we settle our objects (such as persons) and predicates (such as *is bald*).

Definition 2 (Predicates and objects). *Let $P \in PRED$ be a predicate and $x \in OBJ$ be an object. Within a dimensional structure M, a predicate P is a subset of M, written as $||P||^M \subseteq M$ and an object x is an element of M, $x \in M$.*

When obvious, we often skip ^{M} and just write $||P||$ for the simplicity. The core part of this formalization has been partly presented as dimensional structure. But in order to enjoy its expressive power of dimensionality, we need the following function, already known and familiar : *projection.* Thanks to projection, once given a dimensional structure M^i, you can consider another dimensional structure $\Downarrow_i M$ as a representation that you know *less* than the original structure.

Definition 3 (Projection). *Fix $j \in I$. Note $x = \{x_0 \in X_0, x_1 \in X_1, ..., x_j \in X_j, ...\}$. Consider a dimensional structure $M^i = \prod_i X_i$. Pick an arbitrary $j \in I$. A projection $f_j : M \mapsto \Downarrow_j M$ returns $f_j(x) = \Downarrow_j x = \{x_0 \in X_0, x_1 \in X_1, ..., x_{j-1} \in X_{j-1}, x_{j+1} \in X_{j+1}, ...\}$.*

And the other direction (i.e. to add information) is also possible by taking its inverse f^{-1}.

Remark 1. There are many types of projection function. However, for the sake of simplicity, this paper only consider the straightforward and obvious one f_j which just eliminates and ignores an axis specified by X_j. Some possible and expressive projections could be more *distorted*. It seems to make possible to capture more fine-ground distinctions among disagreements and whatsoever.

Even if negation in an original model M is classical (i.e. $||\neg\phi|| = M \setminus ||\phi||$), projection would give give the overlapping area where both ϕ and $\neg\phi$ hold. This is not problematic for us. Rather, we will utilize this behavior to model sorites paradox. Let us formalize this concept.

Definition 4 ($||?||$). *Define $||?_\phi|| = ||\phi|| \cap ||\neg\phi||$. $||?_\phi|| \cap ||\phi|| = ||\phi||$, if not empty. If $x \subseteq ||?_\phi||^M$, then $M \models \phi(x)$* and *$M \models \neg\phi(x)$.*

We will check this behavior of $||?||$ in the following subsection. But roughly, $||?||$ behaves as if it is an wildcard for a truth value gap and/or glut. Finally, we reach at the truth condition for predicates.

Definition 5 (Evaluation). *$M \models P(x)$ (read: "x is P" is true under the interpretation M) if and only if $x \subseteq ||P||$.*

Remark 2 (Subset makes true.). Note that this semantics adopts subset $\subseteq$ instead of $\in$ as most standard framework do.

2.3 Demonstration

Employing the formalization above, let us demonstrate how to capture vague predicates. Under my dimensional perspective, the source of vagueness is not single anymore. Rather, it has two directions where vagueness comes from: *abusence* and *abundance*. What follows observes these two origins.

Case 1: When we know too little. First, let us see a case of vagueness whose origin is *absence* of information. In other words, we consider a kind of vagueness when we do *not* know things *enough*. As an example, let us evaluate baseball players. In baseball, players are evaluated in many perspectives such as hitting, fielding and running (and even more detailed and splitted; For instance, hitting can be at least splitted into hitting power and hitting average. However, let me keep it simple for the sake of simplicity). There have been (finite in reality but still quite) many indices suggested to grade how well they are with respect to each perspective (and sometimes combinations of them). But suppose, reasonably, our ultimate goal is always to distinguish and pick a good player as a whole among non-good ones. Let us simplify the components of baseball here: we only consider three major aspects labelled fielding, hitting, and running. Assume, also, that we have reliable methods to categorize players into good or bad for each of these three elements. Hence, our players in question are evaluated *without* vagueness for each tool. All of them are evaluated as, so to speak, "certainly" good or "certainly" bad for each perspective. See the following table to grasp its dimensinoal structure. Read + as "is good" while − as "is non- good".

M	P1	P2	P3	P4	P5	**D1**
Fileding	+	+	+	?	-	
Hitting	+	-	?	+	-	
Running	+	+	+	?	-	
D2						

$$\Downarrow_{D^2}$$

$\Downarrow$ **M**	P1	P2	P3	P4	P5
In total	+	?	+	+	-

Now, we are ready to see each player's evaluation as a whole. We use $\Downarrow M$ for such total evaluation. Look at the player $P1$. S/he is, without question, a good baseball player, which performs well in every aspect (being considered here) of baseball. $M \models \textsc{good}(P1)$ for $P1 \subseteq ||\textsc{good}||^M$. Our formal setting further confirms this obvious reasoning. Our projection function tells a "digest" version of this evaluation to player $P1$. Tracing any information given to this player in 2-dimensional structure (good for fielding, good for hitting, good for running) into 1-dimensional structure (good or bad as a whole), the projection tells $\Downarrow M \models \textsc{good}(P)$ (and $\Downarrow M \not\models \textsc{bad}(P)$). Similarly, another player $P5$ is a horrible player whose performance is horrible in any way. Formally, $\Downarrow M \models \textsc{bad}(Q)$ (and $\Downarrow M \not\models \textsc{good}(Q)$).

How about the evaluation of player $P2$? S/he is good for some (viz. fileding and running), but bad for the other (hitting). Looking at the original model M, s/he is *neither* good nor bad because $P2 \not\subseteq ||\textsc{good}||^M$ and $P2 \not\subseteq ||\textsc{bad}||^M$. Following the projection function and the generated model by it, s/he turns out to be a *both* good *and* bad player. Here comes vagueness with *insufficient* information with less dimensions: when we evaluate player P2 with enough perspectives (in this case, with $M = D1 \times D2$), there is no vagueness with respect to the player P2. Each element in this original model $M = D_1 \times D_2$, written in the form of a tuple $(player, tool)$ is clearcut divided into good and bad. But once you shortcuts some information, while some are still clearcut distinguishable, others become vague.

Case 2: when we know too much. We have just observed how lacking information leads to vagueness. Some may think, then, *adding* information to consider recovers from vagueness. In some case, it works. For instance, in the case of boaderline player $P2$ above, s/he is vague if we see a limited set of information of $\Downarrow M$. However, if you add a information dimension i.e. see in $M = D1 \times D2$, her/his vagueness disappear. S/he is good or bad in each tool.

However, adding new perspectives does not promise to resolve vagueness. It could expose vagueness hidden in the original and relatively poor consideration. For example, look at the player $P3$. Let us imagine her/him as a designated hitter and you (or anyone) have not seen him running and fielding. In overall judgement, s/he is evaluated as good. But to reach it, s/he does not need to be a 5 tool players like P1. Even if we do not know for some tools (i.e. valuation gaps), we, following my projection function, conclude that s/he is a good player as a whole.

In this scenario, adding extra information (by adding extra axes/dimensions) exposes vagueness. In fact, P3 is neither good nor bad in the original model M because $P3 \not\subseteq ||\textsc{good}||^M$ and $P3 \not\subseteq ||\textsc{bad}||^M$. Hence, we may make vagueness not only by considering little information but also too much information.

3 SORITES PARADOX REVISITED

The previous baseball examples have demonstrated how dimensional understanding works and both directions (not only eliminating but also adding information) cause vagueness. Let us return to our original issue. How does my dimensional view see sorites?

3.1 Reconstruction and reformalization

From our dimensional perspective, the number of hair could be an important but still just one of many indices (or dimensions) which matter when you evaluate whether a given object in question is bald or not. For further details, confirm the following observations.

1. First, the number of hair is *not sufficient* for evaluating baldness. In other words, when we judge whether or not somebody is bald, we need further extra viewpoints other than the of hair. Knowing the number of hair itself does not determine the truth value of predicate "is bald".
2. Second, the number of hair is *not necessary* for the baldness judgement neither. In fact, we often think and believe that someone is bald without knowing the exact number of her hair.
3. Third and finally, we need a better set of viewpoints to specify less obvious ones being located between obvious ones such as zero hair at all or 2,000,000 of hair. This point just claims that to judge non-extreme one "somewhere between extreme cases" is more difficult and requires a more polished (sufficient and non-confusing) information compared to extreme cases such as no hair at all.

Following these observations above, let us build a countermodel M. As mentioned already, the number of hair (labelled $D^{\#}$) itself is not the determining dimension for baldness. Formally, let $D^{\#} = ||?||^{D^{\#}}$. Extra dimensions are needed to evaluate baldness. We have a plenty of candidates for dimensions which give evaluation on our non-informative (with respect to baldness) $D^{\#}$. As an instance, let us here consider "from whose perspective" labelled in D^*. The resulting model $M = D^* \times D^{\#}$ is shown below.

M =

$\mathbf{D}^* \times \mathbf{D}^{\#}$	0	1	2	...	l	l+1	...	m	m+1	...	n	n+1	...	2,000,000	$\mathbf{D}^{\#}$
You	+	+	+	...	+	+	...	+	+	...	+	+	...	-	
Me	+	+	+	...	+	+	...	+	+	...	+	?	...	-	
Her	+	+	+	...	+	?	...	?	-	...	-	-	...	-	
$\mathbf{D}^*$															

$$\Downarrow_{D^*}$$

$\Downarrow_{\mathbf{D}^*}$ **M**	0	1	2	...	l	l+1	...	m	m+1	...	n	n+1	...	2,000,000	$\mathbf{D}^{\#}$
Us	+	+	+	...	+	+	...	+	?	...	?	?	...	-	

Based on the countermodel M defined, we can block the paradox in the two directions: knowing too little and knowing too much.

Knowing too little. Knowing just the number of hair for each person (told by $D_{\#}$) does not conclude whether the person in question is bald. Such judgement requires further set of information which specifies impact on which such number puts impact on bald/non-bald evaluation. For example, let us consider an extra information index: "according to whose impressions (of three persons: you, me and her)", labeled D^*. Then, understand the sorites scenario as happening in the projected model $\Downarrow M$. [1]

The apparent cases (x_0 with 0 hair and $x_{2,000,000}$ with $2,000,000$) are considered as obviouse cases even in the model with more limited information. In the digestive evaluation, there is no vagueness for these two obvious ones because we initiated the sorites paradox with the assumpions that $\Downarrow M \models B(x_0)$ "anyone agrees that a person with no hair at all is bald" and $\Downarrow M \models \neg B(x_{2,000,000})$ "anyone agrees that a person with 2,000,000 hair is not bald". Projection tells the total evaluations of such apparent cases wihtout changing their evaluations since in $\Downarrow M$, $x_0 \subseteq ||B||^{\Downarrow M}$ and $x_{2,000,000} \subseteq ||\neg B||^{\Downarrow M}$.

However, there are many ones less obvious in between. There may be something like one labeled x_n which is a borderline case for the evaluation in $\Downarrow M$. Why? If we consider a view of another person (her), x_n is not bald for sure. But your own perspective tells it is. The projection function returns a contradicting evaluation in $\Downarrow M$: x_n is both bald and not bald. In a way, in order to evaluate baldness of such in-betweens like x_n, we need further dimension D^* specifying

[1] There would be further dimensions composing such imporessions D^*.

"according to whom". You can clearly, without vagueness, whether x_n is bald or not *for another person's perspective and criteria*. But when you are asked in a more digestive version, the answer would become rather vague: she is bald in a way and not bald in another way.

Knowing too much. Let us observe the opposite case. According to this dimensinoal structure given in the previous table, x_{l+1} is bald in total. There is no vagueness at the digesitve evaluation. However, once you upgrade your information by seriously checking other dimensions (in our scenario, considering what everyone thinks of), the threshold becomes less sharper. In fact, the digestive evaluation for x_{l+1} concludes without knowing what the opinion of *her*. They just rely on your first impression/opinion. The projection tells $\Downarrow l+1 \subseteq ||B||^{\Downarrow M}$ while the same object $l+1$ in a different model with further information D^* $l+1 \not\subseteq ||B||^M$.

3.2 Understanding paradox

Noticing points mentioned just above, we can understand a sorites paradoxical situation better now without touching any assumptions nor reasoning system.

Obviously non-bald case: $M \models \neg B(x_{2,000,000})$
Obviously bald case: $M \models B(x_0)$
Tolerance Principle: There are *no* x_n and x_{n+1} such that $M \models B(x_n)$, $M \not\models \neg B(x_n)$, $M \models \neg B(x_{n+1})$ and $M \not\models B(x_{n+1})$

Unwelcome conclusion avoided: A person with 2,000,000 hairs does not have to be bald.

Let us fix the information dimensions $M = D^{\#} \times D^*$ as settled above. According to M, a person with no hair at all is bald (and, satisfying classical logic, is not non-bald) as the assumptions tell. To write formally, $M \models B(x_0)$ and $M \not\models \neg B(x_0)$. Also, another apparent case with many enough hair is non-bald. $M \models \neg B(2,000,000)$ (and, fulfilling classicality, $M \not\models B(2,000,000)$). These assumptons just require M to be acceptably good as a set of information for evaluating these very objects: a person with 0 hair and a person with $2,000,000$ hair, obvious ones for evaluating baldness. As already warned, it does *not* promise this information stricutre also provides sufficient set of information for other objects. In particular, upon such M, we can construct what is *undetermined* for baldness like x_{l+1}, x_m and x_{m+1} : $M \not\models B(x_{l+1})$ and $M \not\models \neg B(x_{l+1})$.

Alternatively, instead of the original M, looking at the squeezed $\Downarrow M$, we can construct "contradicting" ones like x_{m+1} such that $\Downarrow M \models B(x_{m+1})$ and $\Downarrow M \models \neg B(x_{m+1})$.

Neither of them bothers the original setting for the paradox, including the troublesome *tolerance* principle. These contradicting x_{m+1} or unspecified x_{l+1} are independent of tolerance. Recall tolerance states "any small change like pulling one hair does not matter for the baldness". Formally written, it just says $\models B(x_n) \to B(x_{n-1})$ and $\models \neg B(x_n) \to \neg B(x_{n-1})$. However, what happens in contradicting and undetermined objects is not such a dramatic change. What happens is a change from obviously bald to not obviously so. As for undetermined $l+1$, from obviously bald l, it does not suddenly change into non-bald. Rather, it is still bald in a perspective (namely from $\Downarrow M$) but becomes less obvious. $M \not\models B(l+1)$ but it does *not* mean that $M \models \neg B(l+1)$, keeping the tolerance. An overlapping $m+1$ or n does not violates tolerance either. Let us see the difference between m and $m+1$. Looking through M, $M \not\models B(m)$ and $M \not\models B(m+1)$, having nothing to do with tolerance! Even if we see things from less detailed perspective (i.e. $\Downarrow M$), it does not irriate any assumption of the sorites. You can bridge obviously bald ones such as x_0 and obviously non-bald ones such as $x_2, 000, 000$ by less obvious ones like n which needs further dimensions to determine whether $B(n)$ or $\neg B(n)$.

4 SUGGESTIONS CONNECTED

We have introduced the formal components of my semantics. Recall that my purpose is not defeat options already available but embrace them in an unified framework. Now, towards this end, I will demonstrate connections lying between my dimensional understanding and the previous attempts to this paradox. One of the main merits of my semantics is to offer a formal *platform* for existing (and sometimes conflicting) explanations. In other words, this formalization can accommodate several variants, which describe each philosophical solution. Under my dimensional regeme, we can express philosophical disagreement itself (i.e. why and how they disagree with each other) in a unified framework. Among many, let us observe connections to a few popular accounts: supervaluationist's and epistemicist's.

4.1 Supervaluationists dimensionalized

Supervaluationists like Fine would accept a dimensional standpoint as the general background theory. Recall that supervaluationists are characterized by allowing truth-value gaps. In addition, Fine proposes *specification space*, a set which assigns "supervalue" to a sentence whose truth (non-super) value is undetermined.

What overlaps between supervaluationists' semantics and my dimensionalists' semantics? First of all, we all accept truth-value gaps (e.g. whether the player S is good on fielding). Notice that specification space is just a supervaluationists' name for adding an extra dimension to the structure for consulting further detailed information. Their formal desirata of truth value *gaps* are well written in our dimensional framework either by having too much information which conflicts within itself or too less information which does not offer enough evidence. In both cases, we hesitate to assign a rigid truth value.

Moreover, the operator which plays a central role in supervaluationists' theory is defined in our dimensinoal terminology. For one thing, *supervaluationists* such as Fine followed by Keefe [3] would adopt this semantics because imposing dimensions makes possible to describe semantic differences which they expressed by the operator D (read "definitely").

Definition 6 (Definitely). *$M \models D(Px)$ if and only if there is $j \in I$ such that $\Downarrow_j M \models Px$ and $\Downarrow_j M \not\models \neg Px$.*

Let me specify the location of supervaluationists with respect to dimensionalists. Supervaluationism is a mere special version of dimensionalists. Fine settles some constraints over specification spaces such as *stability* (once fixed a truth-value, it cannot become vague anymore) and *completability* (there exists a way to reach the *complete* specification space, which eliminates vagueness for any proposition). I am inclined to be more pessimistic for such constraints to actually or necessarily govern our vague terms or concepts. In fact, our dimensional formalization aims to capture less ideal aspects of our processing information such as *misunderstanding* or being *deceived*s. For example, we are not so sure of stability because we can add an extra dimension which just confuses the judgement already fixed by the information we have already got. Completaility is also questionable for a similar reason.

However, for the time being, I do not mean to disagree with supervaluationists' program as a whole. Rather, I am being satisfied by confirming that this very argument (whether such conditions are justified or not) can be discussed using my dimensional formal setting.

4.2 Epistemicists dimensionalized

Interestingly, one of the supervaluationists' rivals *epistemicists* (e.g. [6]) would also accept my semantics as a natural formalization on their epistemic account, which claims that vagueness is due to our epistemic shortage or *ignorance* that we do not know the sharp threshold between P and non-P. In my dimensional picture, information given to us is limited in the number of dimensions.

Epistemicists claim that we are just too dumb to know where sharp thresholds are placed. This epistemic incapability can be expressed in my dimensional semantics. What epistemicists seem to have in their mind is paraphrased as lack or absence of information: too little dimensions at our hands. The size of dimensions is not enough. As the example of player R tells, we can eliminate such kind of vagueness due to the shortage of information by adding a satisfying set of extra information. Still, the dimensional framework can do more. Our epistemic ignorance does not only occur in shortage but in abundance. Our dimensional framework also makes possible to describe many variants of why and how we are so stupid. We often face or create (sometimes unnecessary) vagueness when we think or consider too much. To be open to any new information could welcome unwanted sources of information such as prejudice or fantasy.

One of the merit of accepting epistemicists approach is to save classical logic. Epistemicists keep this virtue in our dimensional framework just by saying that we are too dumb to have working (i.e. not only sufficient and but also non-confusing) set of dimensions even though the classical logic governs the ontic reality. Our epistemic ability is often limited; we may not know some important dimensions or we may hold fishy dimensions.

5 CONCLUSION:

We have presented a new semantics which features dimensions. We have provided formal definitions for its core part with several intuitive examples as its desiderata. The aim was to give a better account to the puzzling scenario known as sorites paradox, which contains vague predicate. What I have done is not to solve the paradox by specifying and revising problematic assumptions or reasoning among them. Instead, our dimensional view offered a common ground or *platform* of account which can accommodate many philosophical solutions as their particular versions.

Our formalization, I admit, is far from perfect and complete. Future tasks include:

- to check other suggested explanations and solutions can be embedded into this dimensional framework; especially multi-value approach.
- to confirm translations between this and them.
- to seek and expand other types of projection function, with a hope to capture "distorted" information from the same source (e.g. illusion, deceived)

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